Quantifiers in Word Grammar

Dick Hudson, 31 August 2000

The following are some notes on how quantifiers should/might be treated in WG, which were prompted by discussions with Matthias Trautner at ESSLLI in Birmingham last week.

1. Joint and distributed readings
The analysis of quantifiers rests on three simple principles which govern the interaction of sets and individuals in a WG semantic analysis (and possibly more generally). The first is concerned with joint and distributed readings for words that denote sets.

A. Distributed sets
A word which refers to a set of individuals may provide either that set or its member as its contribution to the higher semantic structure - i.e. it may be given a 'distributed' reading (instead of the simpler 'joint' one).

For example,
(1) an essay by two students
This can refer to a single essay written jointly by two students or to two different essays, each by one student - the distributed reading. In the second reading, therefore, it refers to a set of two essays, where the number of essays reflects the number of students. More usually, perhaps, this set-hood is expressed overtly by the use of a plural noun *essays*. Thus the distributive reading of (1) has the same semantics as one reading of (2).
(2) (two) essays by two students
Fig 1. shows the two structures for this example.

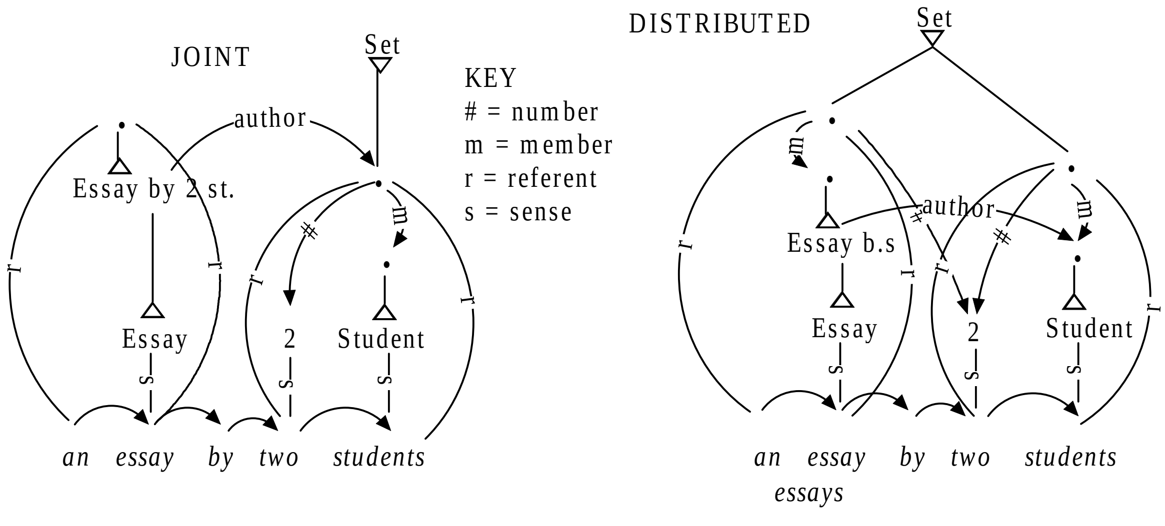

Figure 1

I don't think this principle allows any 'mixed' interpretations - i.e. a mixture of joint and distributed readings. E.g.
(3) two essays by five students
I think this example only has two readings, according to the number of essays:

(4)	a	joint:	total essays = 2
	b	distributed:	total essays = 10

I.e. it cannot be applied to a situation in which there are, say, six essays, of which two come from one student and the remaining four from one student each. If so, it is easy to explain: the difference is a structural difference, and the structure only allows two possibilities: either the dependent is the set (joint) or it is the set's member (distributed). However, as explained under Principle C below, there is no commitment regarding the number of students

2. Number agreement
Another important principle that emerges from this simple example is that the distributed reading implies 'number agreement' between the two sets - the set defined by the dependent (i.e. in this case the number of students) and the one defined for the parent (i.e. the number of essay-pairs).

B. Number agreement
If the member of a set depends on another node, then the latter is also taken as the member of a set, and the two sets share the same number.

In other words, the dependent set 'projects' its number up onto the higher one. We can generalise beyond the examples given by presenting the two possible ways of using a set in diagrammatic form:

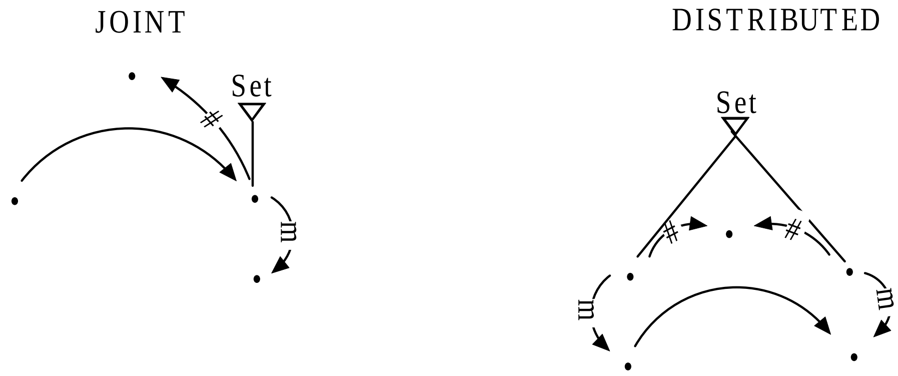

Figure 2

3. Dependents with variable identities
If a word's parent refers to a set, the dependent word may have different referents for each member of the set. For example,
(5) five essays by one student
This can refer to a set of essays which were each written by a different student, but this is only one of many possibilities - for example, it could also be used where two of the essays were written by the same student, and similarly for any number of essays up to the maximum of five. The only requirement is that each essay be written by a single student. This situation is generalised by the next principle:

C. Uncertain identity
A word which refers to a set of individuals may have variable referents for any dependents, but the extent of this variability is uncertain, ranging from total identity (i.e. no variation) to total difference (i.e. no overlap).
The WG analysis for this example is shown in Fig. 3. The crucial part of this diagram is the 'author' node, about which all we know is that it Isa 'student'; there is no commitment on

whether it is the same student for each essay.

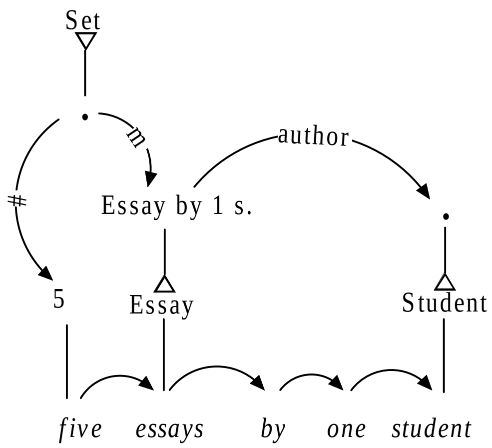

Figure 3

The principle of Uncertain Identity has two important theoretical consequences:

- It shows that sets only project number in one direction, from dependent to parent but not the other way round. This supports the idea that dependencies are fundamental.
- More importantly, perhaps, it calls into question the standard treatment of such examples in the predicate calculus in terms of quantifier scope. The problem with this approach is that it gives a binary structural analysis to what is in fact a continuously variable phenomenon. It handles the two extreme cases well:

(6) a one student: $\exists$s, $\forall$e, by (e,s)

b five students: $\forall$e, $\exists$s, by (e,s)

But for intermediate cases we have to choose arbitrarily between the two. What this analysis does not reveal is that one of these structures subsumes the other: the first is just a special case of the second in which all the students happen to be the same.

4. Verb + one set-denoting dependent

The discussion so far has only involved noun phrases. We now turn to sentences, the usual stamping ground of predicate logic. How do these principles help in the semantic analysis of an example like the following?

(7) One student wrote three essays.

Three essays may be taken jointly or distributively. The joint reading is straightforward, and is shown in Figure 4; it denotes a single event in which a student writes a triple of essays - possibly alternating among them, but in any case without any attempt to distinguish three different events.

The distributive reading is more complicated, as it recognises a distinct writing event for each essay. It projects the number 3 up to the parent, so the sense of *wrote* after combining with *three essays* is a set of three distinct events of writing one essay. This combines with *one student* to give another set which, in the usual way, Isa the first. By inheritance it has 3 members, and its member Isa ‘writing one essay’, but more specifically it Isa ‘one student writing one essay’. This does not however commit us to a single student; by the principle of Uncertain Identity, there could be up to three distinct students. This reading is also shown in Figure 4.

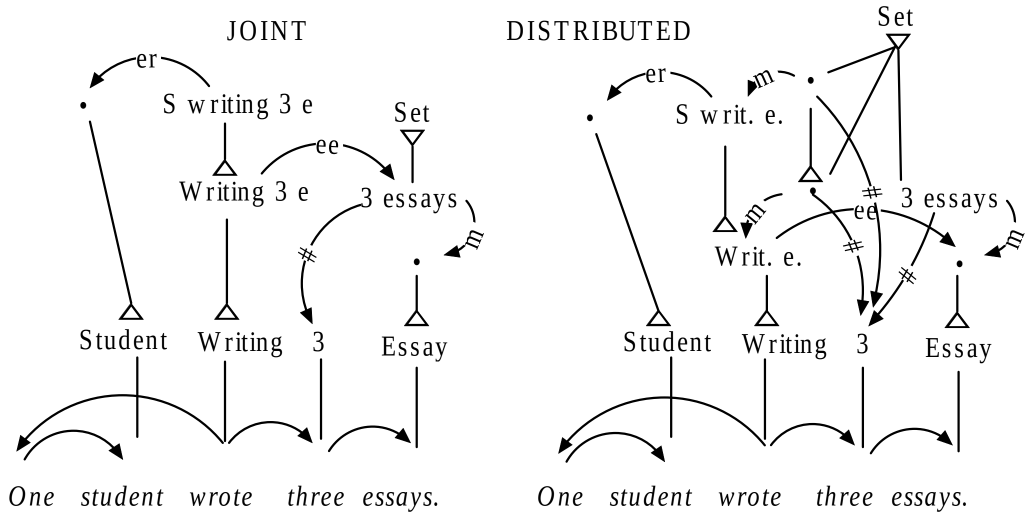

Figure 4

In short, this analysis allows us to distinguish two readings of the sentence which correspond to the two quantifier scopes:

(8) a Joint: $\exists$s, $\exists$e, 3(e) & wrote (s, e)
‘There is a student *s* and a set of 3 essays *e* such that s wrote e.’

b Distributed: $\exists$e, $\forall$x, x$\in$e, $\exists$s, 3(e) & wrote (s, e)
‘There is a set of 3 essays *e* such that for each member of *e*, *x*, there is a student *s* such that s wrote x.’

The difference between the two approaches lies in:

- the mechanism for making the distinction: the left-right order of symbols or the structure of a network;
- the assumed mapping between syntax and semantics: global (all quantifiers stacked up at the left) or local (effects of quantification located locally);
- the locus of the differences: in a special structure (quantifiers only) or integrated with other aspects of the analysis;
- the place given to events: none or a central one.

5. Verb + two set-denoting dependents

We now consider a somewhat more complicated example, in which there are two NPs that denote sets.

(9) Two students wrote three essays.

The principle of Distributed Sets allows both of these NPs to have either a joint or a distributed reading, and the principle of Uncertain Identity allows a dependent which is affected by a distributed reading to have variable reference. The interpretations predicted by this analysis are therefore as follows:

(10) a Joint + Joint: a single pair of students jointly wrote a triple of essays. (2 students, 3 essays, one event of 2 students writing 3 essays)

b Distributed + Joint: a single pair of students each wrote a (presumably distinct) triple of essays. (2 students, 3-6 essays, 2 events of 1 student writing 3 essays)

c Joint + Distributed: a (possibly different) pair of students jointly wrote each of a single triple of essays. (2-6 students, 3 essays, 3 events of 2 students writing 1 essay)

d Distributed + Distributed: a single pair of students each wrote each of a (presumably distinct) triple of essays. (2 students, 3-6 essays, 6 events of 1 student writing 1 essay)

The first two of these readings are shown in Figure 5 (which, for simplicity, omits ‘referent’ links and the Set supercategory).

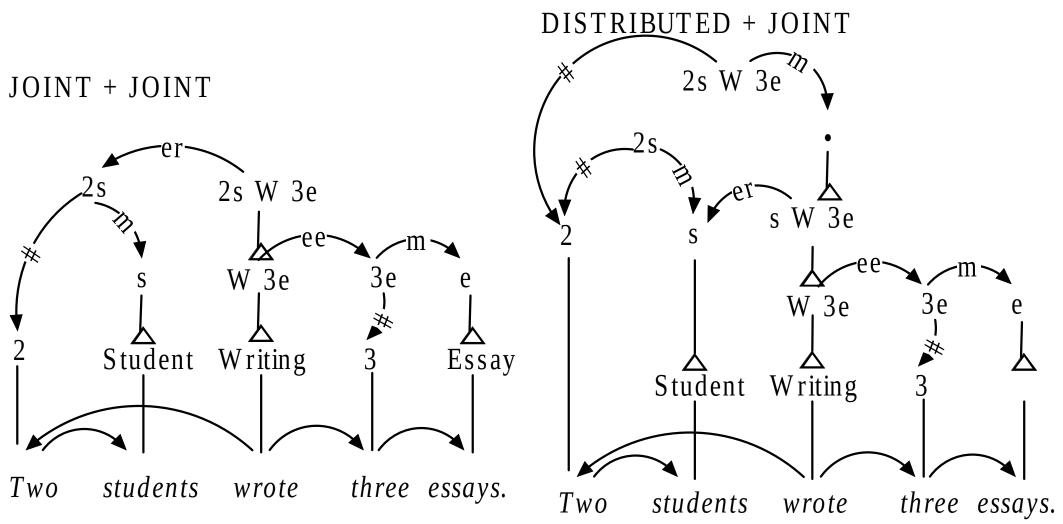

Figure 5

The most important difference between these two structures is the number agreement in the second, shown by the arrows converging on ‘2'. This convergence results from the number of *two students* projecting up onto the sense of *writing* labelled ‘2s W 3e’ (i.e. ‘Two students writing three essays’), a set of events each of whose members is an example of ‘s W 3e’, i.e. one student writing three essays.

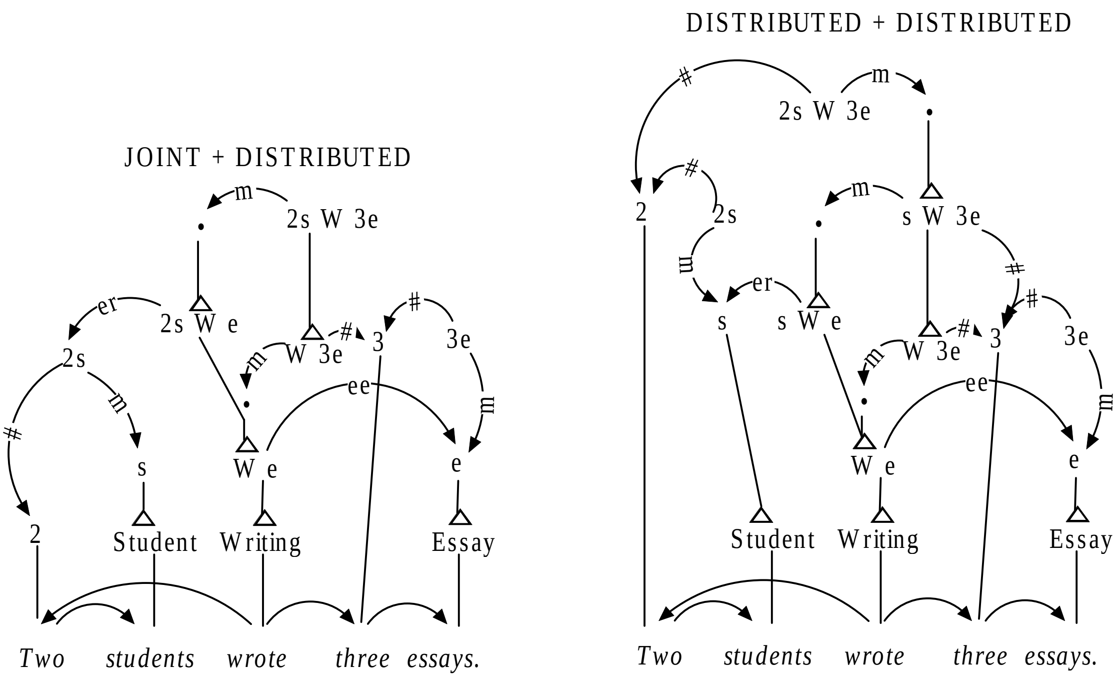

Figure 6

The two other readings of the same sentence are shown in Figure 6. The structure for JOINT + DISTRIBUTED is very similar to the one for DISTRIBUTED + JOINT. One added complication is that it includes two sets of events: ‘W 3e’ (3 events of writing one essay) and

‘2s W 3e’ (3 events of two students jointly writing one essay). The relationship between these two events is Isa, so the more specific one inherits the latter’s number (3).

The fourth structure, DISTRIBUTED + DISTRIBUTED, is the most complicated of all because it includes the added complexities of both the other distributed readings. It includes a simple ‘spine’ of basic events: Writing, ‘W e’ (writing one essay) and ‘s W e’ (one student writing one essay), but these contribute to the definition of three sets of events: ‘W 3e’ (three instances of writing one essay), ‘s W 3e’ (three instances of one student writing one essay) and ‘2s W 3e’ (two instances of three instances of one student writing one essay). It is the last of these that gives the total number of events as six, and it is of special interest because it is a set of sets - its member is an instance of the set ‘s W 3e’.

6. Subsets

The above discussion is intended to prepare for the analysis of quantifiers in the next section. The main point of the discussion was to establish the need for sets as well as individuals, and to lay out the basic principles of Distributed sets, Number agreement and Uncertain identity. In this section we move nearer to the familiar territory of quantifiers by introducing the ‘subset’ relationship (not previously recognised in WG).

The subset relationship is regularly expressed in English by the preposition OF, whose head denotes the subset and whose dependent denotes the superset - e.g. *two of the students* refers to a subset of the students. (This is often called the ‘partitive construction’, but I have no suggestions for partitives with mass nouns, such as *some of my money*.) This pattern is easy to represent in WG notation, with the addition of the standard symbol ⊆; the relationship X ⊆ Y will be shown by an arrow labelled ⊆ pointing from X to Y. Figure 7 shows the structure for sentence (11); for simplicity I have assumed a joint reading, though a distributed reading is more plausible in this example. The referent links are dotted to improve intelligibility.

(11) Two of the students passed.

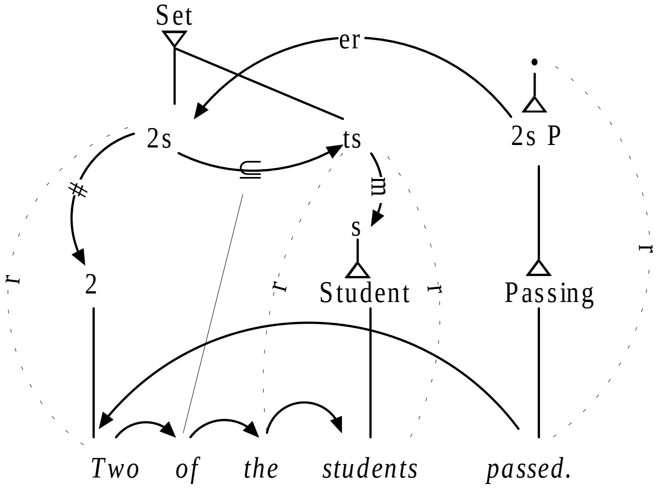

Figure 7

The significance of this example lies in the subset link between ‘2s’ (two students) and ‘ts’ (the students). This guarantees that every member of the former is also a member of the latter, which removes the need for a separate definition of its member. Obviously any numeral could have been used instead of *two*, which raises the question of how the numeral ONE should be analysed, as in *one of the students*. The natural analysis builds on the one just given, but this means that we must recognise one-member sets in the subset pattern, and if there, then presumably also in simpler phrases like *one student*. This assumption seems harmless, and we shall adopt it here, but it means that in principle we should revise Figure 4 (for *One student wrote three essays*) where, for simplicity, we treated *one student* as referring to an individual, not a set.

7. Quantifiers: ALL, SOME, NO/NONE

With the help of the apparatus introduced in the previous sections we can now discuss the standard quantifiers, starting with ALL, SOME and NONE (NO). We start with their use in the partititive construction: *all/some/none of the students*. This can be treated without consideration of the rest of the sentence because the effect of the quantifier is purely local to the two sets concerned (in contrast with that of EACH which will be discussed in the next section). The semantic structures for these three phrases are given in Figure 8.

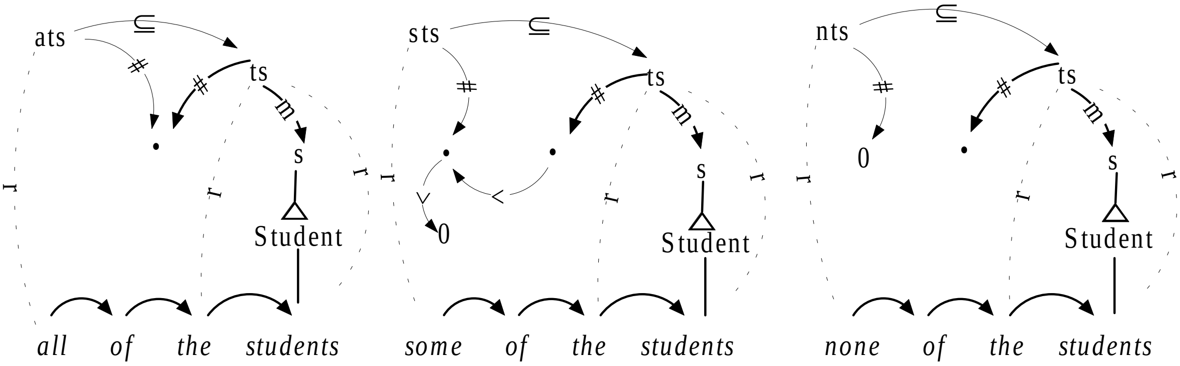

Figure 8

The structures are the same except for their treatment of the number (#) of the subset. For *all of the students* (‘ats’) it is the same as the number of the superset (‘ts’); for *some of the students* it is some figure smaller than the number of the students, but larger than 1; and for *none of the students* it is 0. (The ‘<‘ relationship is the same as the one used for word order, where an earlier word is assumed to have a smaller index number than a larger one.) It should be obvious how to generalise beyond these examples to other uses of the same quantifiers in the partitive construction.

These three quantifiers can also be used without the partitive OF, though the syntactic details vary. ALL can take either a definite determiner (e.g. THE, MY) or a common noun as its complement, but SOME and NONE/NO only take a common noun, and in that case NONE/NO, like some of the possessive determiners, changes shape from *none* to *no*. Thus: *all the students, all students, some students, no students*. As far as their meanings are concerned, however, I assume that the semantic structures in Figure 8 apply to these shorter phrases as well.

One outstanding problem is the syntactic number of *none* and *no*, which may both be syntactically either singular or plural:

(12) a None of the students is/are here.
b No student is here.
c No students are here.

This contrast is relevant to the semantics as well as the syntax, because the usual restrictions apply to verbs which require a set with more than one member, such as DISPERSE or MEET.

(13) a The students/*student dispersed/met.
b No students/*student dispersed/met.

One possible explanation for the effect of syntactic number is that the semantic structure of *no students* is just the same as that of *some students*, but that its (zero) number projects directly up to the parent's semantics (rather as we shall see for EACH); and similarly for *no student* and *a student*.

8. Quantifiers: EACH, EVERY

EACH and EVERY are semantically very similar, though they are syntactically very different:

(14) a Each/*every of the students passed one exam.
b They have each/*every passed one exam.
c Almost every/*each student passed one exam.

These syntactic differences remain to be explained, and it is possible that the explanation is related to some unsuspected difference of semantics, but for present purposes I shall assume that they have identical semantic structures.

The effect of these quantifiers is to force a distributive reading. For example,

(15) a Each/every student wrote an essay.
b A student wrote each/every essay.

In (a) there must be a separate essay-writing event for each student, and in (b) a separate event of a student writing an essay for each essay. In both cases the principle of Uncertain reference means that the other argument co-varies freely - i.e. the essays need not be distinct in (a), nor need the students in (b). This possibility is clearer if we change the verb to READ, since it is easier for more than one person to read the same thing than to write it:

(16) a Each/every student read a book.
b A student read each/every book.

The semantic structures for these examples are given in Figure 9 (in which Set and some referent links are omitted for simplicity).

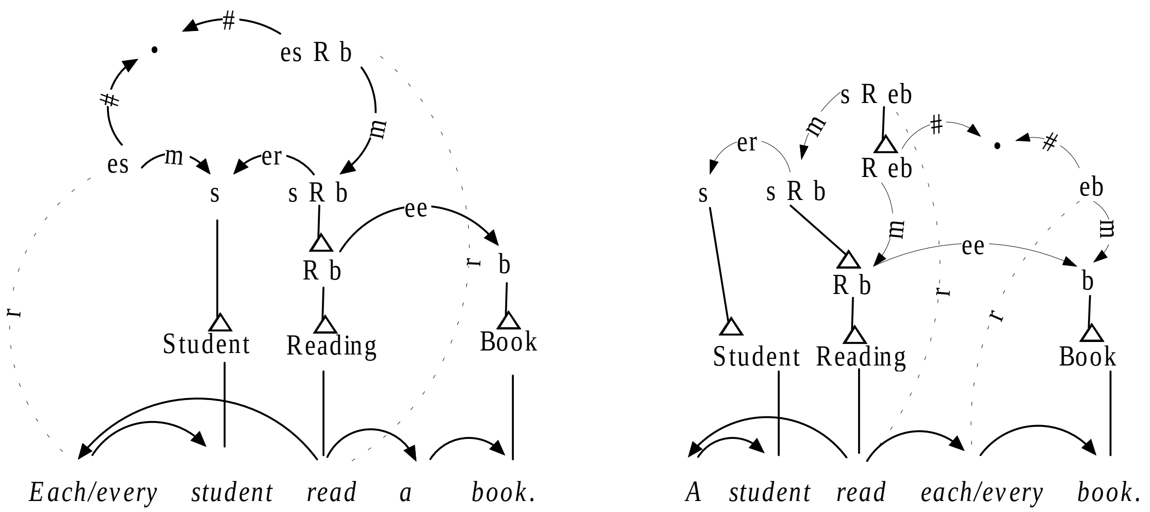

Figure 9

In both structures, EACH or EVERY heads a noun phrase which denotes a set ('es' or 'eb'), in spite of the singular syntactic number. This set supplies its members individually as arguments of the verb, and forces the verb itself to denote a set whose number is the same as that of the noun phrase. All this is normal distributive semantics and requires no further comment.

9. Quantifiers: ANY

The last quantifier to be considered is ANY, which raises extra issues which will have to remain unresolved here. The semantics of ANY is a notorious challenge for basic predicate logic because it hovers between the universal and existential qualifiers. It has some similarities to EACH/EVERY and others to SOME: like the former, it applies (in some sense) to the whole of the presupposed set, but like the latter, it picks out a subset. For instance:

(17) a You can have any cake(s).
b You can have every cake.
c You can have some cake(s).

Sentence (a) offers one cake (singular) or a subset (plural), whereas (b) offers the whole set; and (a) offers a free choice whereas (c) is less liberal, leaving the possibility that the choice could be made by the host (e.g. if followed by *Here it is* or *Here they are*). The element of free choice seems crucial. As I pointed out in EWG (288-9) this is also part of the semantics of the conjunction OR, so I suggested there that ANY may stand to EVERY in the same way that OR stands to AND. In relation to the last set of examples, (a) and (b) could be expanded as follows:

(18) a You can have this cake or this one or this one or .....
b You can have this cake and this one and this one and ...

This basic idea still strikes me as plausible, and underlies the following suggestion.

Suppose the semantic structure of ANY is exactly the same as for EACH or EVERY (notice that, like EVERY, it allows ALMOST: *almost any cake*.), with one difference: the 'selectional type' of the set. Whereas EACH and EVERY have the default selectional type, which is Conjunction - the 'and' relation - ANY has Disjunction - the 'or' relation. In other words, we should use here whatever semantic distinction we find between AND and OR. What the selectional type does, it seems, is to affect the way in which the set relates to the rest of the sentence. Disjunctive semantics is more restrictive than conjunction, because it allows only distributive readings. For example, *John or Bill wrote it* obviously precludes a joint reading, as does *Did any student write it?* However, unlike conjunctive distributive meanings, disjunctive semantics does not project the set number up to the parent set. Instead it projects the number of each member; so *my parents or my friends* projects the number of each of these sets, and *any student* projects a different number from *any students*.

As in any area of linguistics, one thing leads to another and the analyst eventually has to draw a line. This is where my line is at present, with the semantics of coordination on the other side. Maybe one day I shall cross the line (again) and extend the analysis.

10. Summary

In this analysis I have shown how the semantic structures of WG already accommodate sets, and how they can be slightly expanded to accommodate joint and distributive readings. With these small changes it seems to be relatively easy to give a semantics for the standard quantifiers, though there are some loose ends which I am aware of (and also, no doubt, others that I haven't noticed). The analysis looks radically different from standard predicate calculus logic, and I believe that in some respects it rests on distinct (and incompatible) assumptions. I hope it will become clearer in due course whether these differences are real as well as apparent, but meanwhile I think I have at least shown how WG treats some of the phenomena to do with quantifiers.